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Graph Contraction Technique as an Alternative Approach to Solving Balanced Transportation Problem

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Abstract

In this work, we present an alternative method, namely, Graph Contraction Technique for solving the transportation problem which is a special class of Linear Programming Problem. We represented the transportation problem as a bipartite graph and solved it iteratively. To illustrate the method, two numerical examples are solved and the obtained solutions are compared with those obtained via Vogel approximation method. The present method yields the same initial basic feasible solutions of the problem. However, the present method is found to be very easy to understand; use and implement compared to the Vogel approximation method and can be applied on real life transportation problems by the decision makers.

Keywords: Optimization; Graph; Transportation; Spanning tree; Network flow.

1. Introduction

Individuals, families as well as industries are practically faced with economic optimizations such as cost minimization of non-economic items that are vital to their existence. Transportation Problem (TP) which is one such optimization problem common to business organizations is a particular class of linear programming, associated with day-to-day activities in our real life and mainly deals with logistics. It helps in solving problems involving distribution and transportation of resources from one place to another. The concept is primarily concerned with the optimal (best possible) way in which a product produced at different factories or plants (called supply origins) can be transported to a number of warehouses, wholesalers, distributors or customers (called demand destinations). For instance, a firm produces goods at m different supply centres, labelled as $S_1, S_2, \dots, S_m$. The demand for the goods is spread out at n different demand centres labelled $D_1, D_2, \dots, D_n$. The problem of the firm is to get goods from supply centres to demand centres at minimum cost in such a way that the total supplies from all Sources are sufficient to meet the demands at all the destinations. The transportation problem is an important linear programming model that arises in several contexts and has deservedly received much attention in literature. It was first formulated by Hitchcock [1], and was independently treated by Koopmans [2]. However, the problem was solved for optimum solutions as answers to complex business problems when Dantzig [3] applied the concept of Linear programming in solving the transportation model using the simplex method. Since then the transportation problem has become the classical common subject in almost every textbook on operation research and mathematical programming. The common methods used to obtain an initial feasible solution of the problem include: the Northwest-corner method (NWCM), Least-cost method (LCM) or Vogel approximation method (VAM). Finding an initial basic feasible solution using Vogel approximation method was the work of Renifeld and Vogel [4]. In general, the Vogel's approximation method usually tends to produce an optimal or near optimal initial solution. Several researches in this field determined that Vogel produces an optimum solution in about 80% of the problems under test, according to Rekha [5] and Vivek [6]. However, the method is hard to comprehend, highly technical to handle and time consuming, involves high cost of implementation, etc. Aljanabi and Jasim [7] proposed a new approach for solving transportation problem using modified kruskal algorithm. The approach was bias to graph theory while being supported by the kruskal algorithm for finding minimum spanning tree. Recently, Akpan and Iwok [8] worked on the transportation problem in the shipment of cable troughs for an underground cable installation from three supply ends to four locations at a construction site where they are needed; in which case, they sought to minimize the cost of shipment. The problem was modeled into a bipartite network representation and solved using the Kruskal method of minimum spanning tree. Their result showed that the cost obtained in shipping the cable troughs under the application of the method, which was AED 2,022,000 (in the United Arab Emirate Dollar), was more effective than that obtained from mere heuristics when compared.

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1.1. Statement of the Problem

Description of a classical transportation problem can be given as follows: A certain amount of homogeneous commodity is available at a number of sources and a fixed amount is required to meet the demand at a number of destinations. Then finding a method that is simple, easy to understand and cost effective for use in obtaining an optimal schedule of shipment of the commodity with the satisfaction of demands at each destination is the main goal of the problem.

1.2. Aim

The aim of this work is to establish the graph contraction technique as an alternative approach to solving balanced transportation problem.

1.3. Objectives

i. To represent the general transportation problem with bipartite graph.
ii. To obtain a feasible solution of the problem using graph contraction technique.
iii. To verify the scalability of the technique for solving any transportation problem.

2. Methods

2.1. Mathematical Formulation

Let us consider the m-plant locations (sources) as $S_1, S_2, \dots S_m$ and the n-retail Depots (destination) as $D_1, D_2, \dots D_n$ respectively. Let $a_i \geq 0$, $i = 1,2,3, \dots m$, be the amount available at plant S_i. Let the amount required at depot D_j be $b_j \geq 0$, $j = 1,2,3, \dots n$. Let the cost of transporting one unit of product from S_i to D_j be C_{ij}, $i = 1,2, \dots m$, $j = 1,2,3, \dots n$ If $x_{ij} \geq 0$ is the number of units to be transported from S_i to D_j, then the problem is to determine x_{ij} so as to minimize the objective function (z)

$$Min \sum_{i=1}^{i=m} \sum_{j=1}^{j=n} C_{ij} x_{ij} \qquad (1)$$

Formula (1) is subjected to two constraints namely:
Supply constraints:

$$\sum_{j=1}^{j=n} x_{ij} \leq a_i \; (i = 1,2, \dots, m) \qquad (2)$$

Demand constraints:

$$\sum_{i=1}^{i=m} x_{ij} = b_j \; (j = 1,2, \dots, n) \qquad (3)$$

Since all the x_{ij} must be non-negative, the above equations need to satisfy the following additional restriction called the non-negativity constraint:

thus, $x_{ij} \geq 0 \; (i = 1,2, \dots, m; j = 1,2, \dots, n)$

Remark: The set of constraints represents $m + n$ equations in m X n non-negative variables.
This problem can be represented by a Table as shown in Table 1.

Table-1. A typical Transportation Problem Tableau

From \ To	D_1	D_2	...	D_n	Supply a_i
S_1	C_{11} x_{11}	C_{12} x_{12}	...	C_{1n} x_{1n}	a_1
S_2	C_{21} x_{21}	C_{22} x_{22}	...	C_{2n} x_{2n}	a_2
. .	. .	. .	... ...	. .	. .
S_m	C_{m1} x_{m1}	C_{m2} x_{m2}	...	C_{mn} x_{mn}	a_m
b_j	b_1	b_2	...	b_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

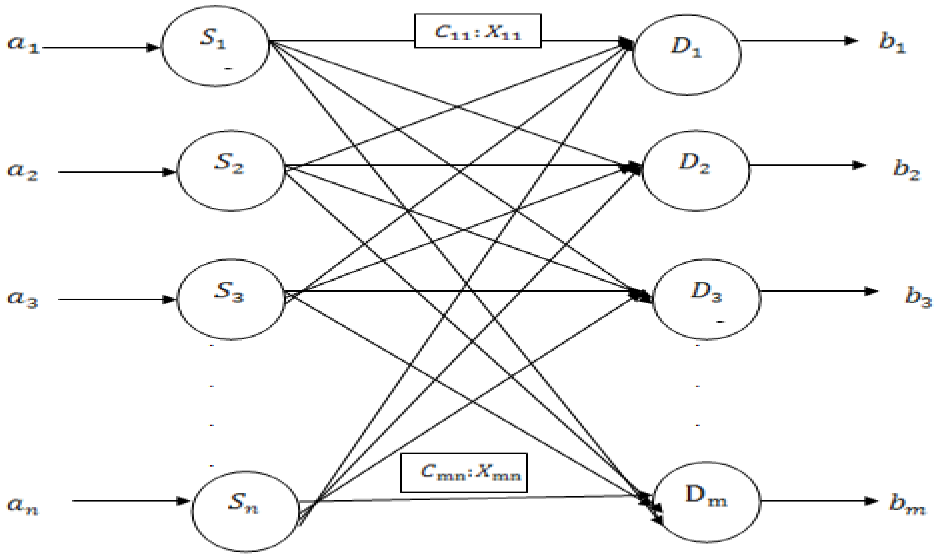
Source: [7].

The graph that appears in the transportation problem is called bipartite graph, or a graph in which all vertices can be divided into two classes, so that no edge connects two vertices in the same class. A property of bipartite graphs is that they are 2-colourable, that is, it is possible to assign each vertex a “colour” out of a 2-element set so that no two vertices of the same colour are connected. We apply our knowledge of spanning trees, network flows and matching to study minimum-cost network flow. To simplify the problem, consider networks whose underlying graphs are complete bipartite. All edges go directly from suppliers to demands and have finite capacity and known per unit cost. We refer to the supply vertices as Sources and the demand as Destinations and edge (i, j) as the link from each source S_i to each destination D_j. There is a cost $C(i, j)$, the unit cost, charged for shipping an item on edge (i, j). The goal is to find a routing of all the items from Sources to destinations that minimizes the transportation costs. This optimization problem is appropriately called the Transportation Problem. It is one of the first optimization problems studied in operations research. In general, warehouse i has a supply of size $S(i)$ and store j has a demandof size $D(j)$. We assume that the total supplied summed over all warehouses, are adequate to meet the demand at all the stores. A solution to the transportation problem specifies the amount $X(i; j)$ to ship the goods from each source S_i to each destination D_j which is minimum.

2.2. Network Representation of the Problem as Bipartite Graph

The network representation of the problem is given as Figure 1.

Figure-1. Graphical Representation of a Typical Transportation Problem



Source: [9]

In operations research, obtaining the optimum solution of the transportation problem is achieved in two phases. The first phase is concerned with finding a basic feasible solution. This is a solution which satisfies all the constraints (including the non-negativity constraint) present in the problem. Different techniques and algorithms are used such as:

i. Northwest-corner method,
ii. Least-cost method and
iii. Vogel approximation method.

The present method depends on graph theoretic approach namely, graph contraction technique. Graph contraction is a technique for implementing recursive graph algorithms, where on each iteration, the algorithm is repeated on a smaller graph contracted from the previous step.

2.3. Algorithm for Graph Contraction Technique

The proposed technique consists of the following steps:

Step 1: Convert the Transportation Tableau into Bipartite Graph.

Step 2: Select the edge with the least unit cost.

Step 3: Assign as much allocation as possible to this edge depending on quantities at source and demand whichever is less.

Step 4: Delete the source or demand vertex depending on which one is satisfied first.

Step 5: Repeat from step 2 until there is no edge left.

3. Results

The following examples are considered to study the correctness, effectiveness and scalability of the present technique.

3.1. Example 1

Sun-Ray Transport Company ships truckloads of grain from three silos (sources) to four mills (destinations). The supply (in truckloads) and the demand (also in truckloads) together with the unit transportation costs per truckload on the different routes are summarized in Table 2. The unit transportation costs, C_{ij} (shown in the northeast corner of each box) are in hundreds of dollars. The model seeks the minimum-cost shipping schedule between the silos and the mills.

Table-2. Data for Transportation Problem Example 1

Destination Source	D_1 (\$)	D_2 (\$)	D_3 (\$)	D_4 (\$)	Supply (a_i)
S_1	10	2	20	11	15
S_2	12	7	9	20	25
S_3	4	14	16	18	10
Demand (b_j)	5	15	15	15	50

Source: Hamdy [9]

Let $x_{ij} =$ number of truckloads transported from i^{th}Source $(i = 1,2,3)$ to j^{th} Destination, $(j = 1,2,3,4)$ implying that m = 3 and n = 4.

The problem can be formulated mathematically in the linear programming form as:

Minimize

$$Z = 10x_{11} + 2x_{12} + 20x_{13} + 11x_{14} + 12x_{21} + 7x_{22} + 9x_{23} + 20x_{24} + 4x_{31} + 14x_{32} + 16x_{33} + 18x_{34}$$

Subject to

Capacity constraint:

$$10x_{11} + 2x_{12} + 20x_{13} + 11x_{14} \leq 15$$
$$12x_{21} + 7x_{22} + 9x_{23} + 20x_{24} \leq 25$$
$$4x_{31} + 14x_{32} + 16x_{33} + 18x_{34} \leq 10$$

Requirement constraint:

$$10x_{11} + 12x_{21} + 4x_{31} \leq 5$$
$$12x_{12} + 7x_{22} + 9x_{32} \leq 15$$
$$4x_{13} + 14x_{23} + 16x_{33} \leq 15$$
$$4x_{14} + 14x_{24} + 1634 \leq 15$$

Non-negativity constraint:

$$x_{ij} \geq 0$$

The above problem has 7 constraint equations and 12 decision variables (i.e. $(n+m)$ and $(n \, \mathrm{x} \, m)$ respectively).

The network representation of the problem as a bipartite graph is as in Figure 2 below

Figure-2. Graph of Problem Example 1

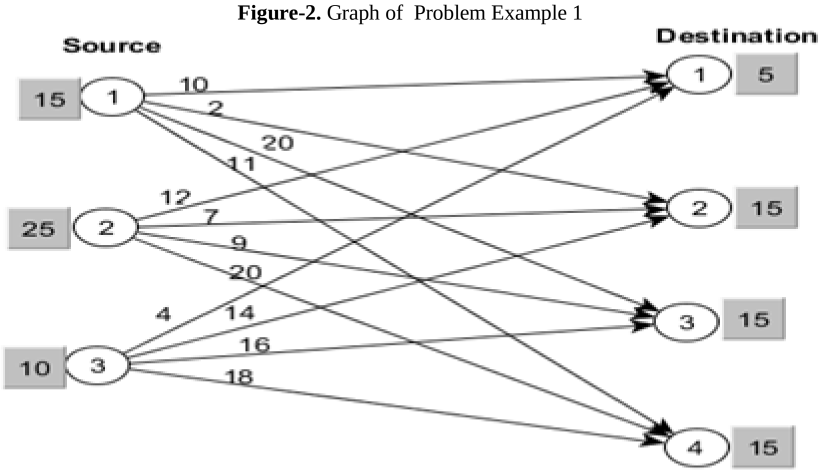


Figure-3. Graph of Iteration 1 (after allocating 15 truckloads to edge S_1D_2 at $\$2, i.e. \$30$)

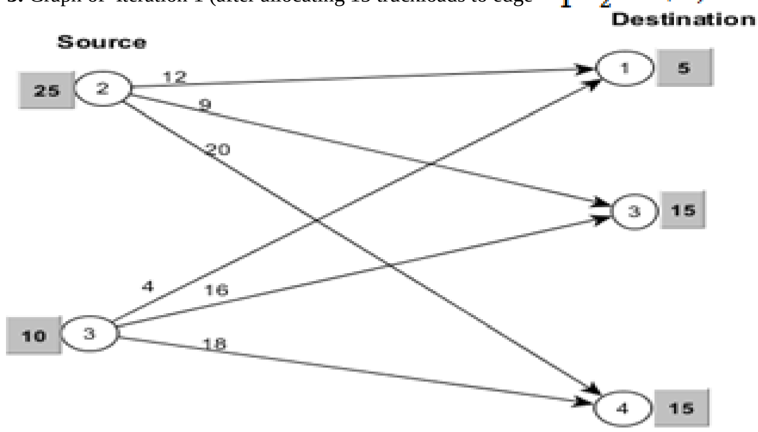


Figure-4. Graph of Iteration 2 (after allocating 5 truckloads to edge S_3D_1 at $\$4, i.e. \$20$)

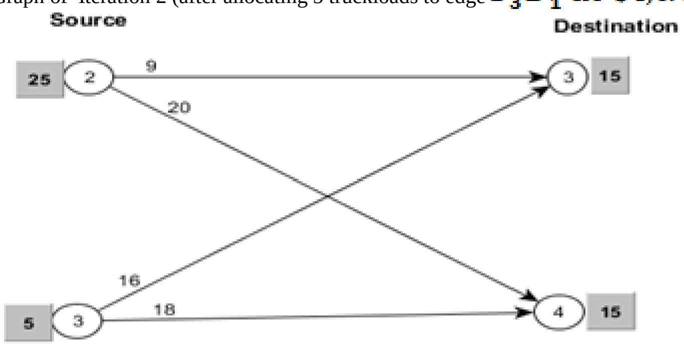


Figure-5. Graph of Iteration 3 (after allocating 15 truckloads to edge S_2D_3 at \$9, $i.e.$ \$135)

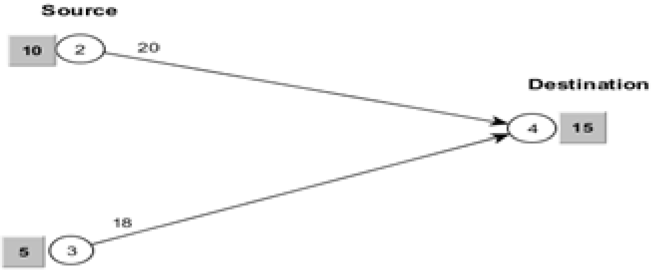
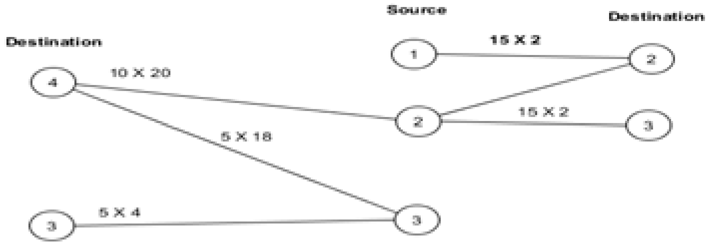


Figure-6. Minimum Spanning Tree of Problem Example 1



From Figure 6 we have that

$$x_{11} = 0, \quad x_{12} = 15, \quad x_{13} = 0, \quad x_{14} = 0, \quad x_{21} = 0, \quad x_{22} = 0, \; x_{23} = 15, \; x_{24} = 10, \; x_{31} = 5, \quad x_{32} = 0, \; x_{33} = 0, \quad x_{34} = 5$$

Thus the objective function,

$$Z = 2x_{12} + 9x_{23} + 20x_{24} + 4x_{31} + 18x_{34}$$

$$= 30 + 135 + 20 + 200 + 90$$

$$= 475$$

Table-3. Result of Problem Example 1

S/N	Edge	Cost (\$)	(Qty)	Amount (\$)
1	S_1D_2	2	15	30
2	S_3D_1	4	5	20
3	S_2D_3	9	15	135
4	S_2D_4	20	10	200
5	S_3D_4	18	5	90
Total Cost				\$ 475

3.2. Example 2

Mahindra manufacturers with facilities in various places have manufacturing plants in Kolkata denoted by letter K, Bangluru by letter B, and Mumbai by letter M. On a normal working day, the usual daily production for these three plants is 50 for Kolkata plant, 40 for Bangluru and 60 for Mumbai plant. In addition, this company has three warehouses located in Vadodra denoted by letter V, Chakan denoted by letter C and Nagpur by letter N. These warehouses have a daily demand of 20 units for Vadodra, 95 units for Chakan and 35 units for Nagpur. Table 4 gives the summary of shipping cost per unit.

Table-4. Data for Transportation Problem Example 2

Destination Source	D_1 (Rs)	D_2 (Rs)	D_3 (Rs)	Supply (a_i)
S_1	6	4	1	50
S_2	3	8	7	40
S_3	4	4	2	60
Demand (b_j)	20	95	35	150

Source: Rekha [5]

Let x_{ij} = number of units of manufactured goods transported from i^{th} Source $(i = 1,2,3)$ to j^{th} Destination, ($j = 1,2,3$) implying that m = 3 and n = 3.

The problem can be formulated mathematically in the linear programming form as:

The network representation of the problem as a bipartite graph is as in Figure 7 below.

Figure-7. Graph of Problem Example 2

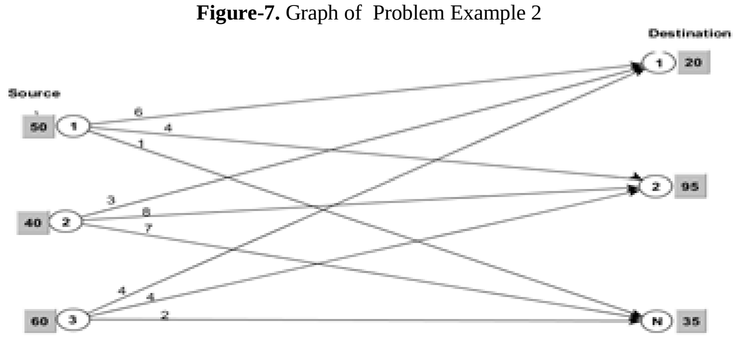


Figure-8. Graph of Iteration 1 (after allocating 35 units of the manufactured goods to edge S_1D_3 $at\ Rs1, i.e.\ Rs35$)

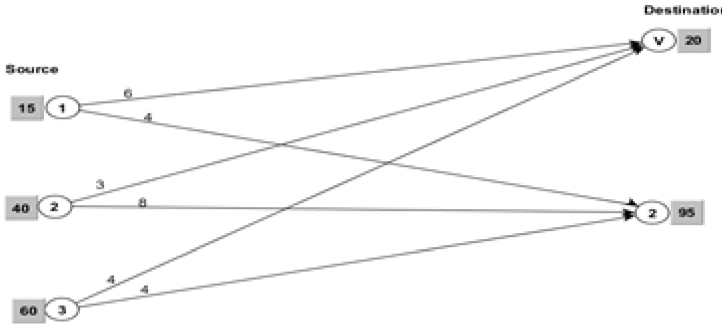


Figure-9. Graph of Iteration 2 (after allocating 20 units of the manufactured goods to edge S_2D_1 $at\ Rs3, i.e.\ Rs60$)

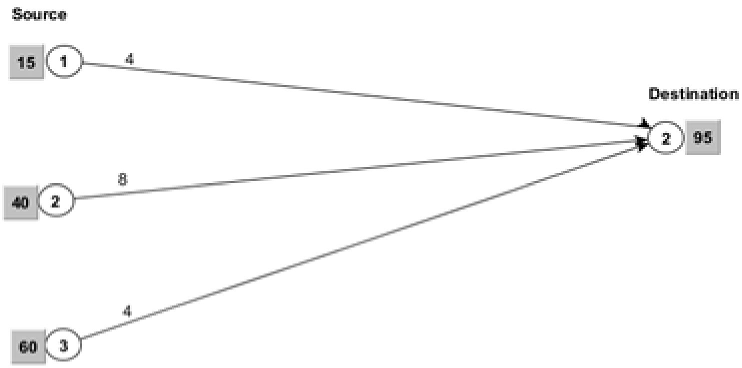
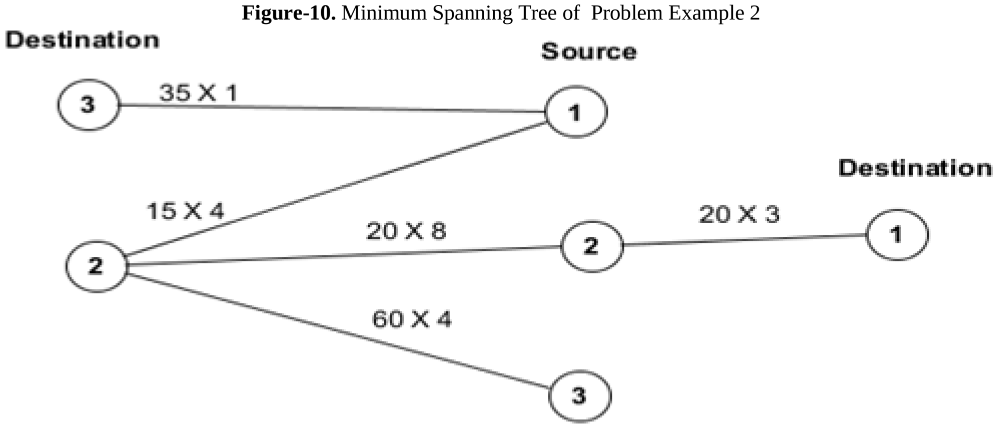


Figure-10. Minimum Spanning Tree of Problem Example 2



From Figure 10 we have that

$x_{11} = 0,\quad x_{12} = 15,\quad x_{13} = 35,\quad x_{21} = 20,\quad x_{22} = 20,\quad x_{23} = 0,\quad x_{31} = 0,$
$x_{32} = 60,\quad x_{33} = 0,$

Thus the objective function,

$$Z = 4x_{12} + x_{13} + 3x_{21} + 8x_{22} + 4x_{32}$$
$$= 60 + 35 + 60 + 160 + 240$$
$$= \text{Rs } 555$$

Table-5. Result of Problem Example 2

S/N	Edge	Cost (Rs)	(Qty)	Amount (Rs)
1	S_1D_3	1	35	35
2	S_2D_1	3	20	60
3	S_1D_2	4	15	60
4	S_2D_2	8	20	160
5	S_3D_2	4	60	240
Total Cost				555

3.3. Tabulation of Results

To illustrate the comparative efficiency, the solutions obtained using Vogel's Approximation Method (VAM) by the two researchers and the present method, Graph Contraction Technique (GCT), are shown in Table 6 below.

Table-6. Summary of Solutions Obtained by Both Methods

Problem	Source	Solution by	Solution by
Example		(VAM)	(GCT)
1.	Hamdy [9]	$475	$475
2.	Rekha [5]	Rs 555	Rs 555

4. Discussion

The presented approach for finding the initial basic feasible solution of the transportation problem is based mainly on using graph contraction technique (GTC) for determining the most cost effective shipment schedule. The algorithm of the approach is detailed with two suitable numerical examples. In each of the cases, all the edges between sources and demands were sorted in an ascending order according to the weights (costs of unit delivery between sources and demands) in an array and starting from the first element of the array which represents the absolute minimum cost, either the source vertex with all its outgoing edges if this source is satisfied is deleted or the targeted demand with all its incoming edges if this demand is satisfied is deleted or even both. This process is repeated until a minimum spanning tree is achieved. Clearly, the technique can be used in different transportation models and gives faster convergence criteria since it is based mainly on reduction of the graph after each iteration. The examples cover only balanced transportation model. Table 6 above gives the analysis of the results obtained using both methods which produced same values.

5. Conclusion

In this thesis we have used an alternative method to solve balanced transportation problem that have earlier been solved by different researchers using Vogel's Approximation Method (VAM). From the investigations and the results given above, we found that Vogel's Approximation Method and the Present Method give the same results for all balanced transportation problems implying that like VAM, it has the merit that it produces an optimal solution in eighty percent of all cases. However, the method presented here is simpler in comparison with Vogel's Approximation Method (VAM). It is very easy to understand, involves simple calculations and thus saves time and can be easily applied to find the initial basic feasible solution for the balanced transportation problems.

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