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Is Schrodinger Equation Describing A Turbulent Flow?

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Abstract: In this article we give a Navier-Stokes like equation with complex variables starting from Schrodinger equation. Two emerging quantities, alpha – describing the force density- and Omega-describing the current density rotation- are found. The flow processes two dimensional characteristics. A definition of pressure arises. The terms in this equation are well separated in real and imaginary parts. The article might be said to give some justification in favour of the ether. What is primary in all these is the description of the velocity in terms of the gradient of an unknown quantity, the wave function.

Keywords: Navier-Stokes; Turbulence; Schrodinger equation; Fluid; Force.

1. Introduction

Quantum mechanics have been associated with a lot of mysticism. One could mention a simple example. In Dirac formalism the operator alpha dagger is used. Now, the dagger is one of the five holy items of Sikh Hindus which all start with the letter K. Now, the lettering of the shells in atoms again goes as KLMN and starts with K. Certainly K must represent a constant but our research has not proceeded thus far. However, it could be mentioned that the missing quantity in quantum mechanics and it is missing in the sense that we Newton's law F=ma is not obeyed, is force. We are going to derive a Navier-Stokes equation for the force in fluids in this article and the key statement is that five as a holy number comes in the expression of a Newton in terms of dynes.

$$1\text{N}=10^5\ \text{dyn} \quad \text{Eq(1)}$$

Now, it is the surface tension that is usually measured in dynes (comes from the Greek work dynami=force) and the subject has been dealt with only slightly by Koutandos [1]. The Navier-Stokes for steady flow is given below [2]:

$$\rho\left(\frac{\partial\vec{v}}{\partial t}+(\vec{v}\cdot\nabla)\vec{v}\right)=-\nabla p+\eta\Delta\vec{v}+(\zeta+1/3\eta)\nabla(\nabla\cdot\vec{v}) \quad \text{Eq(2)}$$

The Greek word η is the viscosity coefficient and it's easy to find out that should be replaced by h in quantum mechanics (it has the same dimensions). The Greek letter ρ is density and we should expect it to be proportional to $|\psi|^2$.

The flow we describe is turbulent and so comes the explanation why Schrodinger used the Greek letter psi for the wave function. It is because after it flows the Omega which is describing the rotation of a turbulent flow. In our case it is describing a two dimensional turbulent flow the equation for which is [3]:

$$\frac{d\vec{v}}{dt}=-\nabla\left(\frac{p}{\rho}\right)-\nu\nabla\times\vec{\Omega} \quad \text{Eq(3a)}$$

In equation (3) ν is the kinematic viscosity which is the viscosity over density.
In order to make better use of equation(3) one may use the fact that it has been proved in literature [4, 5] that a pressure term arising from quantum mechanics formalism, at least the one describing the sum of electric and magnetic pressure is:

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$$P = \frac{|\psi|^2}{N} U \text{ Eq(3b)}$$

The counterpart of Omega is alpha, the letter starting the Greek alphabet and alpha is describing [6] the force density, at least in superconductivity.

$$\vec{\alpha} = \frac{d\vec{F}}{dV} = \vec{J} \times \vec{B} + \frac{|\psi|^2}{N} \nabla U \text{ Eq (3c)}$$

2. Main Part

The rotation of the velocity in quantum mechanics is zero since the wave function psi is single valued and the velocity is defined as the gradient of psi. So we define Omega as:

$$\vec{\Omega} = \nabla \times (\psi * \vec{v}) = \nabla \times \vec{J} \quad \text{Eq(4)}$$

In what follows we are going to need the following vector identities:

$$\nabla \times (\phi \vec{F}) = \nabla \phi \times \vec{F} + \phi \nabla \times \vec{F} \quad \text{Eq(5)}$$
$$\nabla (\vec{F} \cdot \vec{G}) = (\vec{F} \cdot \nabla) \vec{G} + \vec{F} \times \nabla \times \vec{G} + (\vec{G} \cdot \nabla) \vec{F} + \vec{G} \times (\nabla \times \vec{F}) \quad \text{Eq(6)}$$
$$\nabla \times (\vec{F} \times \vec{G}) = (\nabla \cdot \vec{G}) \vec{F} - (\nabla \cdot \vec{F}) \vec{G} + (\vec{G} \cdot \nabla) \vec{F} - (\vec{F} \cdot \nabla) \vec{G} \quad \text{Eq(7)}$$

It may be proved that Omega is real first by using equation(5) into eq(4).

$$i\hbar \vec{\Omega} = i\hbar \nabla \psi * \times \vec{v} = \vec{v} * \times \vec{v} \quad \text{Eq(8)}$$

Then we notice that the external product of two conjugate quantities is imaginary:

$$\vec{A} \times \vec{A}* = (B + i\vec{C}) \times (\vec{B} - i\vec{C}) = 2i\vec{C} \times \vec{B} \quad \text{Eq(9)}$$

Now we are going to calculate the rotation of Omega vector:

$$i\hbar \nabla \times \vec{\Omega} = \nabla \times (\vec{v} * \times \vec{v}) \quad \text{Eq(10)}$$

By substituting Eq(7) into Eq(10) we find:

$$i\hbar \nabla \times \vec{\Omega} = (\nabla \cdot \vec{v}) \vec{v} * - (\nabla \cdot \vec{v} *) \vec{v} + (\vec{v} \cdot \nabla) \vec{v} * - (\vec{v} * \cdot \nabla) \vec{v} \quad \text{Eq(11)}$$

Next we use eq(7) in Eq(11) :

$$i\hbar \nabla \times \vec{\Omega} = (\nabla \cdot \vec{v}) \vec{v} * - (\nabla \cdot \vec{v} *) \vec{v} - 2(\vec{v} * \cdot \nabla) \vec{v} + \nabla |\vec{v}|^2 \quad \text{Eq(12)}$$

So far we have come with some hydrodynamic terms, like the gradient of the pressure and the inertia terms. Next we calculate the viscosity term. As we mentioned it must have the viscosity coefficient in front as well:

$$\begin{aligned} \hbar \psi * \Delta \vec{v} &= -i\hbar \psi * \nabla (\Delta \psi) = i(2m) \psi * \nabla (\psi (E - U)) = \\ &= -i(2m) |\psi|^2 \nabla U + 2\vec{Q} + 2im(E - U) \nabla |\psi|^2 \end{aligned} \quad \text{Eq(13)}$$

$$\vec{Q} = (E - U) |\psi|^2 \nabla \phi = \vec{J} \left(\frac{m}{\hbar} \right) (E - U) \quad \text{Eq(14)}$$

Now, the Q vector is definitely describing a thermal current and this can be seen by taking its divergence plus the fact that it is proportional to the current density times kinetic energy.

In order to integrate eq(13) into eq(12) , first we notice that the first two terms of eq(12) give an imaginary result as they are conjugate. The calculation gives:

$$(\nabla\cdot\vec{v})\vec{v}^{\,*}-\vec{v}(\nabla\cdot\vec{v}^{\,*})=2i\,\mathrm{Im}(\hbar^2\nabla\psi^*\Delta\psi)=\hbar\psi^*\Delta\vec{v}-2\vec{Q}+2mi|\psi|^2\nabla U \quad \text{Eq(15)}$$

As a reminder we write down the Schrodinger equation we used in eq(15),eq(13):

$$-\frac{\hbar^2}{2m}\Delta\psi=(E-U)\psi \quad \text{Eq (16)}$$

Now we have come up with the final equation. We must divide by the factor two as may be easily seen and multiply by the mass to get a better result. It looks like this:

$$-i\frac{\hbar m}{2}\nabla\times\vec{\Omega}+i|\psi|^2\nabla U=m(\vec{v}^{\,*}\cdot\nabla)\vec{v}-\nabla\left(m\left|\frac{\vec{v}}{2}\right|^2\right)+\hbar\psi^*\Delta\vec{v}-\vec{Q} \quad \text{Eq(17)}$$

3. Conclusion

In equation (17) we may recognize the gradient of the kinetic energy as the gradient of a real 3D pressure and an imaginary force density. The effect of turbulence is purely imaginary as expected. The rotation Omega is divided by two because it is two times the angular velocity of a particle which will be spinning in a turbulent flow. What is new is the thermal current Q.

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