Scientific Review
ISSN(e): 2412-2599, ISSN(p): 2413-8835
Vol. 4, Issue. 4, pp: 26-33, 2018
URL: http://arpgweb.com/?ic=journal&journal=10&info=aims

Academic Research Publishing Group

Original Research Open Access

Anekwe's Corrections on the Negative Binomial Expansion

Uchenna Okwudili Anekwe
Department of Physics, University of Science and Technology, Aleiro, Nigeria

Abstract

For so many years now a lot of scientist have used the series of positive binomial expansion to solve that of Negative binomial expansion, positive fractional binomial expansion and Negative fractional binomial expansion which was generated/derived using Maclaurin series to derive the series of Negative binomial expansion, positive fractional binomial expansion and Negative fractional binomial expansion just as it was used to provide answers to positive binomial expansion but fails for All the other expansion due to a deviation made. This Manuscript contains the correct solution/answers to Negative binomial expansions with proofs through worked examples, with other forms of solving Negative binomial expansion just as in the case of Pascal’s triangle in positive binomial expansion, in Negative binomial expansion it is called Anekwe’s triangle and other methods like the combination method of solving Negative binomial expansion.

Keywords: Negative binomial; Expansion; Equation.

1. Introduction

According to Coolidge [1] the binomial Theorem, familiar at least in its elementary aspects to every students of algebra, has a long and reasonably plain history. Most people associate it vaguely in their minds with the name of Newton’s; he either invented it or it was carved on his tomb. In some way or the other it was his theorem. Well, as a matter of fact it wasn’t, although his work did mark an important advance in the general theory.

We find the first trace of Binomial Theorem in Euclid II, 4, “ if a straight line be cut at random, the square on the whole is equal to the square on the segments and twice the rectangle of the segments.” if the segments are a and b this means in algebraic language

$$(a+b)^2 = a^2 + b^2 + 2ab$$

The corresponding formula for the square difference is found in Euclid II, 7, “ if a straight line be cut at random, the square on the whole and that on one of the segments both together, are equal to twice the rectangle contained by the whole and said segment, and the square on the remaining segment,”.

From the formula above it is seen that from the left hand side that if a& b are summed up together and then squared the results obtained must be the same for the right hand side that is a squared plus b squared plus 2 time the product of a & b.

Therefore, if the result for the Negative binomial expansion estimated from the left hand side of an equation is not exactly equal to the Right hand of the equation just as in the case of positive binomial then we can either get an approximate results or a wrong result. With relevant worked examples in this Manuscript shows the exact solution to Negative binomial and the deviations made using the Maclaurin series to obtain the expansion for Negative binomial.

John [2] the negative binomial distribution is interesting because it illustrates a common progression of statistical thinking, and it’s viewed in terms of Counting, Cheap generalization and Modeling over dispersion.

2. Methodology

For so many years, there has been a Mistake made in using the series of positive binomial in finding the expansion of the negative binomial. This material presents new solutions to the series of the Negative binomial using worked examples.

Example 2.1

Expand the following negative binomial

$$(1+x)^{-1}$$

Solution

Using the long division method we’ve

$$(1+x)^{-1} = \frac{1}{1+x}$$

```
          1
1+x  |  1
   -    1+x
   ---------
        -x
```

$$\Rightarrow \frac{1}{1+x}=1-\frac{x}{1+x}$$

$$\therefore\ (1+x)^{-2}=((1+x)^{-1})^{2}=\left(\frac{1}{1+x}\right)^{2}=\left(1-\frac{x}{1+x}\right)^{2}=\left(1-\frac{x}{1+x}\right)\left(1-\frac{x}{1+x}\right)=1-\frac{x}{1+x}-\frac{x}{1+x}+\frac{x^{2}}{(1+x)^{2}}$$

$$\Rightarrow (1+x)^{-1}=1-\frac{x}{1+x}$$

$$(1+x)^{-2}=1-\frac{2x}{1+x}+\frac{x^{2}}{(1+x)^{2}}$$

Studying the series above it is seen that the expansion for negative binomial is in the form of

$$(1+x)^{-n} = a+\frac{bx}{1+x}+\frac{cx^{2}}{(1+x)^{2}}+\frac{dx^{3}}{(1+x)^{3}}+\ldots$$ and not

$(1+x)^{-n} = a + bx + cx^2 + dx^3 + \ldots$ As used mistakenly by many scientist over so many years.
From the series of negative binomial the coefficients can be determined.
Proof:
If

$$\left(1+x\right)^{-n}=a+\frac{bx}{1+x}+\frac{cx^{2}}{(1+x)^{2}}+\frac{dx^{3}}{(1+x)^{3}}+\ldots \qquad (1)$$

$\Rightarrow$ Putting x=0 in equation (1) we've

$$a=1 \qquad (2)$$

Differentiating equation (1) w.r.t x we've

$$-n\left(1+x\right)^{-n}=\frac{b}{(1+x)^{2}}+\frac{2cx}{(1+x)^{3}}+\frac{3dx^{2}}{(1+x)^{4}}+\ldots \qquad (3)$$

Multiplying although equation (3) by $(1+x)^2$ we've

$$-n\left(1+x\right)^{-n+1}=b+\frac{2cx}{(1+x)}+\frac{3dx^{2}}{(1+x)^{2}}+\ldots \qquad (4)$$

Putting x=0 in equation (4) we've

$$b=-n \qquad (5)$$

Differentiating equation (4) w.r.t x we've

$$-n\left(-n+1\right)(1+x)^{-n}=\frac{2.1c}{(1+x)^{2}}+\frac{3.2dx}{(1+x)^{3}}+\ldots \qquad (6)$$

Multiplying although equation (6) by $(1+x)^2$ we've

$$-n\left(-n+1\right)(1+x)^{-n+2}=2!c+\frac{3.2dx}{(1+x)}+\ldots \qquad (7)$$

Putting x=0 in equation (7) we've

$$-n\left(-n+1\right)=2!c$$

$$\therefore\ c=-\frac{n(-n+1)}{2!} \qquad (8)$$

Differentiating equation (7) w.r.t x we've

$$-n\left(-n+1\right)(-n+2)(1+x)^{-n+1}=\frac{3.2.1d}{(1+x)^{2}}+\ldots \qquad (9)$$

Multiplying although equation (9) by $(1+x)^2$ we've

$$-n(-n+1)(-n+2)(1+x)^{-n+3} = 3!d + \ldots \quad (10)$$

Putting x=0 in equation (10) we've

$$-n(-n+1)(-n+2) = 3!d$$

$$\Rightarrow d = \frac{-n(-n+1)(-n+2)}{3!} \quad (11)$$

Substituting equation (2), (5), (8) and (11) into equation (1) we've

$$(1+x)^{-n} = 1 - \frac{nx}{1+x} - \frac{n(-n+1)x^2}{2!(1+x)^2}$$

$$-\frac{n(-n+1)(-n+2)x^3}{3!(1+x)^3} + \ldots \quad (12)$$

Putting $n' = -n$ in equation (12) we've

$$(1+x)^{-n} = 1 + \frac{n'x}{1+x} + \frac{n'(n'+1)x^2}{2!(1+x)^2}$$

$$+\frac{n'(n'+1)(n'+2)x^3}{3!(1+x)^3} + \ldots \quad (13)$$

Where $n' = -n$

OR

$$(1+x)^{-n} =$$

$$1 - \frac{nx}{1+x} - \frac{n(1-n)x^2}{2!(1+x)^2} - \frac{n(1-n)(2-n)x^3}{3!(1+x)^3} + \ldots \quad (14)$$

Note

In the general case such as $(x+a)^{-n}$ we've the expansion to be given as

$$(x+a)^{-n} = \left[x\left(1+\frac{a}{x}\right)\right]^{-n} = x^{-n}\left(1+\frac{a}{x}\right)^{-n}$$

From which $\left(1+\frac{a}{x}\right)^{-n}$ can be expanded using either equation (13) or (14) and then multiplied through by x^{-n}.

Example 2.2.

Find the expansion of the series $(1+x)^{-3}$ and hence compute its numerical value from the expansion when x=1 and when x=3 respectively.

Solution

Using equation (13) above we've ($n' = -3$)

$$(1+x)^{-3} = 1 + \frac{n'x}{1+x} + \frac{n'(n'+1)x^2}{2!(1+x)^2} + \frac{n'(n'+1)(n'+2)}{3!(1+x)^3}x^3 + \ldots =$$

$$1 - \frac{3x}{1+x} - \frac{3(-3+1)x^2}{2!(1+x)^2} - \frac{3(-3+1)(-3+2)}{3!(1+x)^3}x^3 + \ldots = \; 1 - \frac{3x}{1+x} - \frac{3(-2)x^2}{2!(1+x)^2} - \frac{3(-2)(-1)}{3!(1+x)^3}x^3 + \ldots$$

$$\Rightarrow (1+x)^{-3} = 1 - \frac{3x}{1+x} + \frac{3x^2}{(1+x)^2} - \frac{x^3}{(1+x)^3} + \ldots$$

When x=1 we've

$$(1+1)^{-3} = (2)^{-3} = \frac{1}{2^3} = \frac{1}{8} = 1 - \frac{3(1)}{2} + \frac{3(1)^2}{(2)^2} - \frac{(1)^3}{(2)^3} + \ldots = \frac{1}{8} = 1 - \frac{3}{2} + \frac{3}{4} - \frac{1}{8}$$

$$\therefore (1+x)^{-3} = \frac{8-12+6-1}{8} = \frac{1}{8}$$

$\Rightarrow$ When x=1 $(1+x)^{-3} = \frac{1}{8}$

Also when x=3 in the same way we've $(1+x)^{-3} = \frac{1}{64}$

2.2. The Negative Triangle of the Binomial Expansion

Considering the following expansions

$(1+x)^0 = 1$

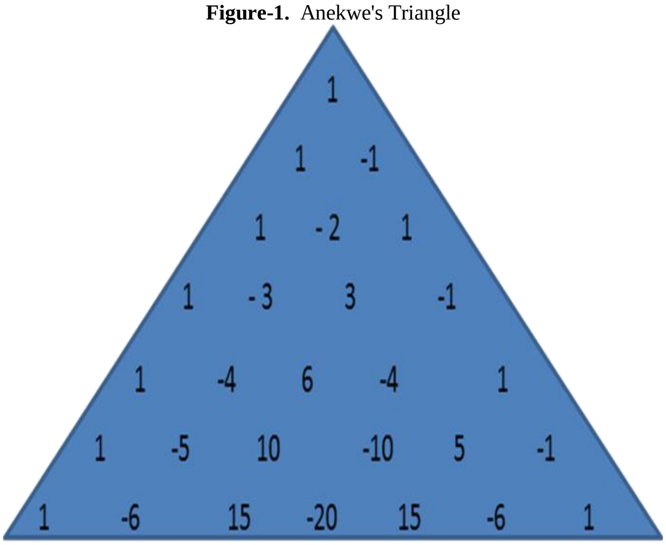
$$(1+x)^{-1} = 1 - \frac{x}{1+x}$$

$$(1+x)^{-2} = 1 - \frac{2x}{1+x} + \frac{x^2}{(1+x)^2}$$

$$(1+x)^{-3} = 1 - \frac{3x}{1+x} + \frac{3x^2}{(1+x)^2} - \frac{x^3}{(1+x)^3}$$

From the expansion above we've

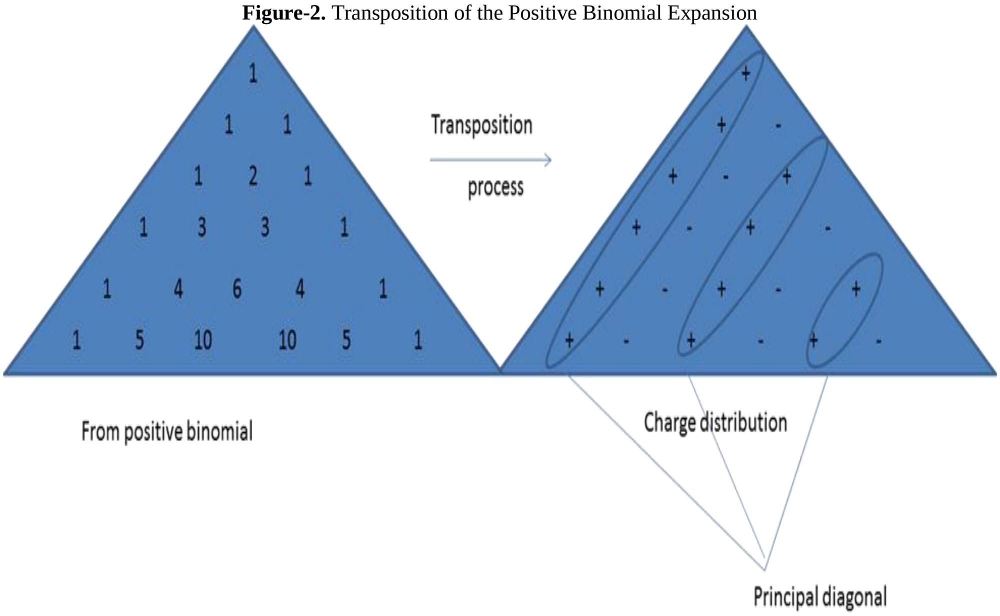
Figure-1. Anekwe's Triangle



The negative triangle of binomial expansion was formed from the series of negative binomial expansion, just like that of the positive triangle of binomial expansion now known as the Pascal's triangle.

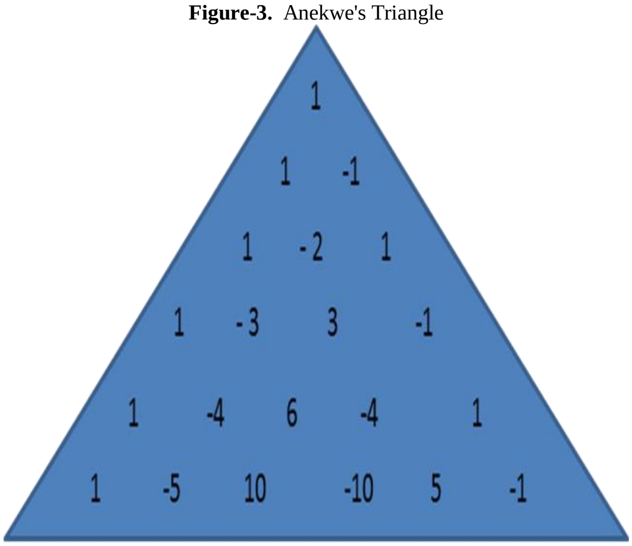
Note that the negative binomial expansion is found the same way the positive binomial is found only that in the negative binomial there is transposition (redistribution) of charges/signs.

Figure-2. Transposition of the Positive Binomial Expansion



After transposition we've

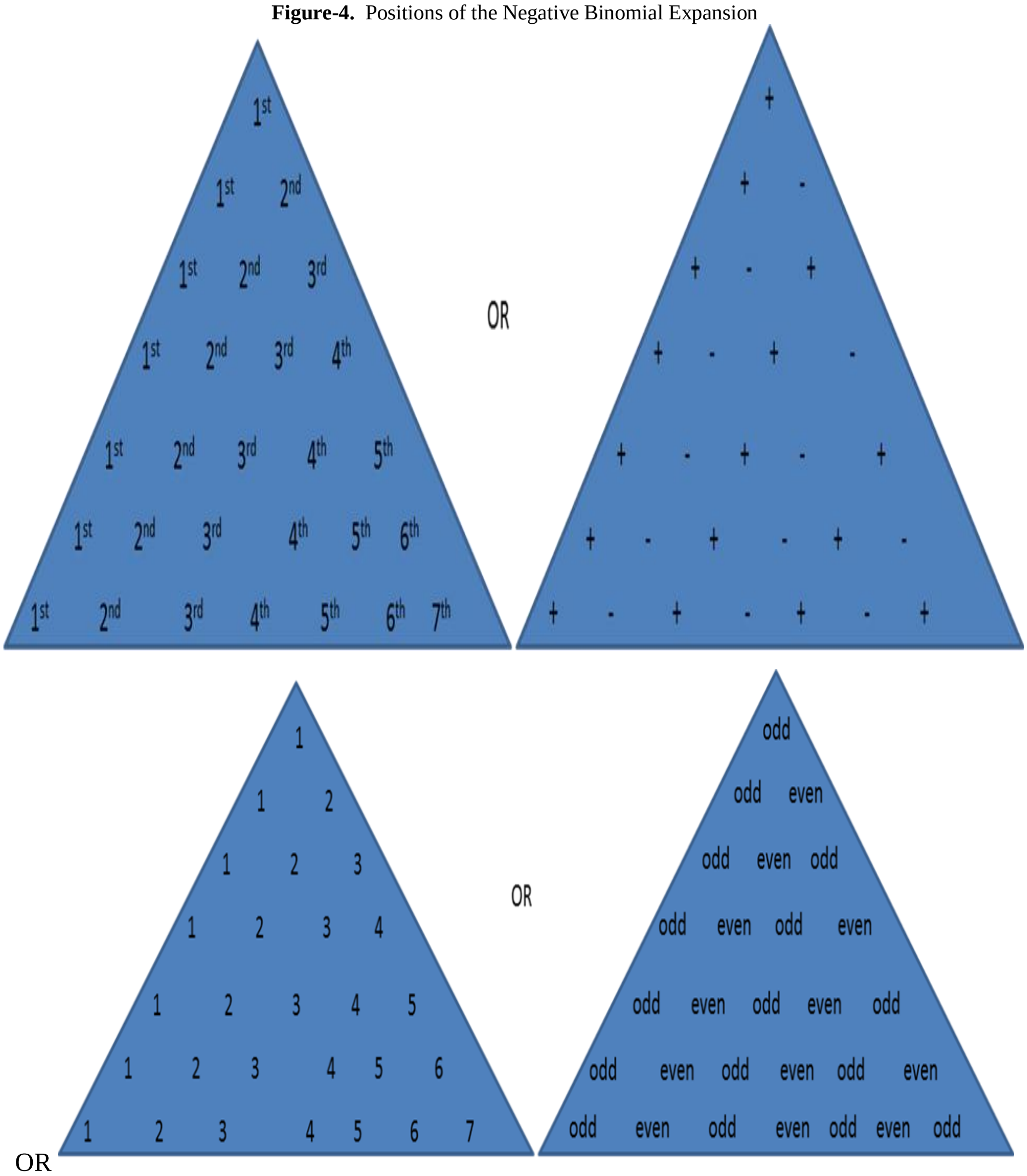
Figure-3. Anekwe's Triangle



Note that after transposition we've the triangle of the negative binomial as the Anekwe's Triangle.

[3-7] For easy understanding note from the charge distribution that all odd principal diagonals are positive while all even diagonals are negative i.e. all odd position in an expansion takes positive sign while all even position after transposition takes negative sign.
I.e. after transposition

Figure-4. Positions of the Negative Binomial Expansion



OR

Note that in negative binomial expansion we've

$$(x+a)^{-n} = x^{-n}\left(1+\frac{a}{x}\right)^{-n} = \frac{1}{x^n}\left[\left(1+\frac{a}{x}\right)^{-n}\right]$$

$$= \frac{1}{x^n}\left[1+\frac{n'\left(\frac{a}{x}\right)}{1+\left(\frac{a}{x}\right)}+\frac{n'(n'+1)\left(\frac{a}{x}\right)^2}{2!\left(1+\left(\frac{a}{x}\right)\right)^2} + \frac{n'(n'+1)(n'+2)\left(\frac{a}{x}\right)^3}{3!\left(1+\left(\frac{a}{x}\right)\right)^3}+\ldots \right]$$

$$= \frac{1}{x^n}\left[1+\frac{n'a}{x+a}+\frac{n'(n'+1)a^2}{2!(x+a)^2}+\frac{n'(n'+1)(n'+2)a^3}{3!(x+a)^3}+\ldots\right]$$

$\Rightarrow$

$$(x+a)^{-n} = \frac{1}{x^n}+\frac{n'a}{x^n(x+a)}+\frac{n'(n'+1)a^2}{2!x^n(x+a)^2} + \frac{n'(n'+1)(n'+2)a^3}{3!x^n(x+a)^3}+\ldots \qquad \text{(x)}$$

Where $n' = -n$

In terms of the negative binomial expansion we've the application of k(constants) from the negative triangle of binomial expansion as

$$(x+a)^{-n} = K_1\left(\frac{1}{x^n}\right)+K_2\left(\frac{1}{x^n}\right)\frac{a}{(x+a)} + K_3\left(\frac{1}{x^n}\right)\frac{a^2}{(x+a)^2}+\ldots \qquad \text{(y)}$$

Example 2.3.

Find the expansion of $(x+a)^{-2}$ from the negative triangle of binomial expansion.

Solution

From the negative triangle of binomial expansion we've the coefficient as (1 -2 1) $\Rightarrow$(K_1=1, K_2=-2, & K_3=1) on substitution into equation (y) above we've

$$(x+a)^{-2} = \frac{1}{x^2}-\frac{2a}{x^2(x+a)}+\frac{a^2}{x^2(x+a)^2}+\ldots$$

Example 2.4.

Find the expansion of $(x+a)^{-3}$ from the negative triangle of binomial expansion.

Solution

From the negative triangle we've the coefficient as (1 -3 3 1)

$$\Rightarrow (x+a)^{-3} = \frac{1}{x^3}-\frac{3a}{x^3(x+a)}+\frac{3a^2}{x^3(x+a)^2}-\frac{a^2}{x^3(x+a)^3}+\ldots$$

2.3. Expansion of the Negative Binomial Using the Combination Method

Looking at the series generated above in equation (X), the
Series can be written in terms of combination given by

$$(x+a)^{-n} = \frac{1}{x^n}\frac{n!}{0!(n-0)!}\left(\frac{a}{x+a}\right)^0$$

$$-\frac{1}{x^n}\frac{n!}{1!(n-1)!}\left(\frac{a}{x+a}\right)^1 + \ldots + \frac{1}{x^n}\frac{n!}{n!(n-n)!}\left(\frac{a}{x+a}\right)^n$$

Example 2.5.

Expand the following negative binomial using the combination method $(x+1)^{-2}$

Solution

$$(x+1)^{-2} = \frac{1}{x^2}\frac{2!}{0!(2-0)!}\left(\frac{1}{x+1}\right)^0$$

$$-\frac{1}{x^2}\frac{2!}{1!(2-1)!}\left(\frac{1}{x+1}\right)^1 + \frac{1}{x^2}\frac{2!}{2!(2-2)!}\left(\frac{1}{x+1}\right)^2$$

$$= \frac{1}{x^2}(1) - \frac{1}{x^2}2\left(\frac{1}{x+1}\right) + \frac{1}{x^2}(1)\left(\frac{1}{x+1}\right)^2 = \frac{1}{x^2} - \frac{2}{x^2(x+1)} + \frac{1}{x^2(x+1)^2}.$$

Find the expansion of $(2x+1)^{-2}$

Solution

$$(2x+1)^{-2} = \frac{1}{(2x)^2}\frac{2!}{0!(2-0)!}\left(\frac{1}{2x+1}\right)^0$$

$$-\frac{1}{(2x)^2}\frac{2!}{1!(2-1)!}\left(\frac{1}{x+1}\right)^1 + \frac{1}{(2x)^2}\frac{2!}{2!(2-2)!}\left(\frac{1}{2x+1}\right)^2$$

$$= \frac{1}{4x^2} - \frac{2}{4x^2(2x+1)^1} + \frac{1}{4x^2}\frac{1}{(2x+1)^2}$$

$$\Rightarrow (2x+1)^{-2} = \left[\frac{1}{4x^2} - \frac{1}{2x^2(2x+1)} + \frac{1}{4x^2}\frac{1}{(2x+1)^2}\right].$$

2.3. Real Life Application of Binomial Theorem

The binomial theorem has a lot of Applications. [8, 9] Some of the applications in real life Situations are:

★ **Computing**

In computing areas, binomial theorem has been very Useful such as in distribution of IP addresses. With Binomial theorem, the automatic distribution of IP Addresses is not only possible but also the Distribution of virtual IP addresses.

★ **Economy**

Economists used binomial theorem to count probabilities that depend on numerous and very distributed variables to predict the way the economy will behave in the next few years. To be able to come up with realistic predictions, binomial theorem is used in this field.

★ **Architecture**

Architecture industry in design of infrastructure, allows engineers, to calculate the magnitudes of the projects and thus delivering accurate estimates of not only the costs but also time required to construct them. For contractors, it is a very important tool to help ensuring the costing projects is competent enough to deliver profits.

2.4. Applications of Binomial Expansion in Physics

The binomial Expansion has other Applications in physics amongst which we've its Applications in

1. Gravitational time dilation.
2. Kinetic energy
3. Electric quadrupole field.
4. Relativity factor gamma
5. Kinematic time dilation.

Other Applications of Binomial Expansions are in:

★ [10] Agriculture in Solving Problems in Genetics.

3. Conclusion

From the worked examples done above we’ve seen the correct solution to Negative binomial expansion and learnt how to solve Negative binomial using Anekwe’s triangle and combination methods together with Negative binomial expansion methods of solving Negative binomial expansion.

References

[1] Coolidge, J. L., 1949. *The story of the binomial theorem*. Harvard University.

[2] John, D., 2009. "Cook Negative binomial distribution."

[3] Godman, A., Talbert, J. F., and Ogum, G. E. O., 1984. *Additional mathematics for West Africa*. Harlow: Longman.

[4] Egbe, E., Odili, G. A., and Ugbebor, O. O., 2000. *Further mathematics published by Africana*. Gbagada, Lagos: FEB Publishers.

[5] Stroud, K. A. and Dexter, J. B., 2013. *Engineering mathematics*. 7th ed. Coventry University, UK.: Palgrave.

[6] Tuttuh, A. M. R., Sivasubramaniam, S., and Adegoke, R., 2016. "Further mathematics project 2."

[7] Uchenna, O. A., 2017. *The foundation to mathematical sciences*. Germany: Lambert Academic Publishing.

[8] 2013. "Real Life Application of Binomial Theorem." Available: https://irsyandirifan.wordpress.com

[9] Carl, R. N., 2015. "Binomial expansion." Available: http://hyperphysics.phy-astr.gsu.edu/hbase/alg3.html

[10] CAI Xiuqing, L. J., ZHUANG Nansheng,, 2016. "Application of the binomial theorem in solving problems in genetics."